

# THE SPEED OF LIGHT AS A DILATON FIELD

Walter Wyss

*Department of Physics*

*University of Colorado*

*Boulder, CO 80309*

Through dimensional analysis, eliminating the physical time, we identify the speed of light as a dilaton field. This leads to a restmass zero, spin zero gauge field which we call the speedon field. The complete Lagrangian for gravitational, electromagnetic and speedon field interactions with a charged scalar field, representing matter, is given. We then find solutions for the gravitational-electromagnetic-speedon field equations. This then gives an expression for the speed of light.

## I. Introduction

We are interested in the concept of physical time. Physical time is defined with the help of a periodical physical system, e.g., an atomic clock. At present we have adopted a universal time currency. But what about local time currencies?: commonly one uses the gravitational redshift as an exchange rate; an atomic clock in Boulder has a higher frequency than a similar atomic clock in Paris. Are there other contributions to this exchange rate? To look for an answer we formulated [1] electrodynamics in such a way that the physical time never occurs. Only the geometric time  $x^o$ , which is related to the physical time  $t$  by  $x^o = ct$  occurs.  $c$  is the speed of light and knowing the geometric time one can recover the physical time. In Special Relativity and also in Einstein's Theory of Gravity only the geometric time occurs; it has the physical dimension of a length. The speed of light only enters if one wants to convert to physical time. The speed of light thus does not have to be a constant, but is related to the concept of physical time. Indeed, in the formulation of electrodynamics, written independently of physical time, the speed of light enters as a scale factor. It can thus be interpreted as a dilaton field [2]. Since this field is related to the speed of light we call it the speedon field. It belongs to the restmass zero, spin zero representation of the Lorentz group. The corresponding elementary agent we called the "speedon." We thus have a trinity of restmass zero gauge particles: speedon (spin zero), photon (spin one) and graviton (spin two).

For interactions with a charged scalar field representing matter each of these gauge fields contribute through their own covariant derivative.

In Chapter II we give the general formulation for gravitational interactions as described by a Lagrange variational principle.

In Chapter III we present the complete interaction between the gravitational field, the electromagnetic field, the speedon field and a charged scalar field representing matter.

In Chapter IV we solve the equations of motion in the absence of the matter field. This gives a solution for the gravitational field and the electromagnetic field in terms of

the speedon field. Turning off the electromagnetic field we find an expression for the pure speedon field and for the corresponding speed of light.

Appendix A describes the scaling of the electromagnetic field and the motivation to introduce the speed of light as a dilaton field.

In Appendix C the gravitational interaction with a restmass scalar field is revisited.

## II. Gravitational Interactions

With the notation in [3] gravitational interactions are given through a variational principle where the action is given by

$$A = \int dx \sqrt{g} R - 2\kappa \int dx \sqrt{g} L(g, \phi) = \int dx L \quad (\text{II.1})$$

and where  $dx$  is the 4-volume element,  $L(g, \phi)$  is the nongravitational Lagrangian, with  $\phi$  any multicomponent field, and

$$\kappa = \frac{8\pi G}{c^2} \quad (\text{II.2})$$

with  $c$  being the speed of light and  $G$  the universal gravitational constant.

Observe that

$$G_o = \frac{G}{c^2} \quad (\text{II.3})$$

has the physical dimension

$$[G_o] = M^{-1}L. \quad (\text{II.4})$$

Throughout,  $M$  = mass,  $L$  = length,  $T$  = physical time,  $Q$  = charge.

Thus  $G_o$  does not depend on the physical time, i.e., the concept of a second, and

$$\kappa = 8\pi G_o \quad (\text{II.5})$$

The action  $A$  with physical dimension

$$[A] = L^2 \quad (\text{II.6})$$

also does not depend on physical time. The physical dimension of the nongravitational Lagrangian  $L(g, \phi)$  is

$$[L(g, \phi)] = ML^{-3}, \quad (\text{II.7})$$

i.e., a mass density.

For the Euler derivative with respect to the gravitational field  $g_{\alpha\beta}$  we get

$$\epsilon(g_{\alpha\beta}) = \frac{\partial}{\partial g_{\alpha\beta}} - \partial_\mu \frac{\partial}{\partial g_{\alpha\beta, \mu}} \quad (\text{II.8})$$

$$\epsilon(g_{\alpha\beta})[\sqrt{g}R] = -\sqrt{g}[R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R] \quad (\text{II.9})$$

and

$$\epsilon(g_{\alpha\beta})[\sqrt{g}L(g, \phi)] \equiv M^{\alpha\beta}, \quad (\text{II.10})$$

where  $M^{\alpha\beta}$  is the so-called gravitational stress tensor.

Introducing  $T^{\alpha\beta}$  through

$$M^{\alpha\beta} \equiv -\frac{1}{2}\sqrt{g}T^{\alpha\beta} \quad (\text{II.11})$$

we then get Einstein's equation

$$R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R = \kappa T^{\alpha\beta} \quad (\text{II.12})$$

Observe that

$$T^{\alpha\beta} = -2\epsilon(g_{\alpha\beta})[L(g, \phi)] - g^{\alpha\beta}L(g, \phi) + 2\Gamma^\nu_{\mu\nu} \frac{\partial L(g, \phi)}{\partial g_{\alpha\beta, \mu}} \quad (\text{II.13})$$

In addition we have the equations of motion for the field  $\phi$

$$\epsilon(\phi)[\sqrt{g}L(g, \phi)] = 0 \quad (\text{II.14})$$

Comments

- 1) The action for gravitational interaction is invariant under the inhomogeneous Lorentz group. In addition it is invariant under the gravitational gauge group [4] and any other gauge group or internal symmetry group.

- 2) The right-hand side of Einstein's equation (II.12) is not arbitrary but is related to  $M^{\alpha\beta}$ , which is an Euler derivative. If one looks at the matter Lagrangian  $L(\eta, \phi)$ , i.e., the Lagrangian  $L(g, \phi)$  where the gravitational field  $g$  is replaced by the Minkowski metric  $\eta$ , then there is the concept of an energy-momentum tensor [5]. The right-hand side  $T^{\alpha\beta}$  in Einstein's equation, evaluated for  $g = \eta$  is exactly equal to the energy-momentum tensor belonging to  $L(\eta, \phi)$  [6]. That this is true for any field is highly nontrivial [7] and is a consequence from the fact that the energy-momentum tensor for any gravitational interaction vanishes identically.
- 3) For dust one usually takes

$$T^{\alpha\beta} = \rho u^\alpha u^\beta, \quad u^\alpha u_\alpha = 1 \quad (\text{II.15})$$

This, however, does not fit in the above scheme; the closest we can get to such a case is by looking at a massless scalar field, i.e., a dilaton field. This field then also has an equation of motion. For mathematical consistency the right-hand side of Einstein's equation should be an Euler derivative.

### III. Graviton, Photon, Speedon and a Charged Scalar Field

Here we give the most general theory of the interactions between a charged scalar field and gauge fields belonging to the rest mass zero [8]. This involves the restmass zero fields of spin 2 (gravity), spin one (electrodynamics) and spin zero (the speed of light). These are all gauge fields and bring their own covariant derivative. The restmass zero, spin zero field, which is a dilaton field, is related to the scaling of the electromagnetic field such that the physical time never enters the corresponding Lagrangian (Appendix A).

We thus have the fields  $g_{\alpha\beta}, L_\alpha, S, \phi^+, \phi$  with the physical dimensions

$$[g_{\alpha\beta}] = 0, \quad [L_\alpha] = MQ^{-1}, \quad [S] = 0, \quad [\phi^+] = [\phi] = 0 \quad (\text{III.1})$$

The action is given by

$$A = \int dx L \quad (\text{III.2})$$

where

$$L = \sqrt{g}R - 2\lambda_o\sqrt{g}L_o(L_\alpha) - 2\sqrt{g}L_o(S) - 2\sqrt{g}L_M(\phi). \quad (\text{III.3})$$

The coupling constant  $\lambda_o$  is

$$\lambda_o = \frac{2G_o}{K_o} \quad , \quad K_o = \frac{K}{c^2} \quad (\text{III.4})$$

with  $K$  being the Coulomb constant.  $K_o$  and  $\lambda_o$  have the physical dimensions

$$[K_o] = MLQ^{-2} \quad , \quad [\lambda_o] = M^{-2}Q^2 \quad (\text{III.5})$$

All these constants do not depend on physical time.

In what follows all indices are raised with the inverse gravitational field  $g^{\alpha\beta}$  and lowered with  $g_{\alpha\beta}$ . The Lagrangian for the electromagnetic field  $\{L_\alpha\}$  is given by

$$L_o(L_\alpha) = \frac{1}{4}F_{\mu\nu}F^{\nu\mu} \quad (\text{III.6})$$

$$F_{\mu\nu} \equiv \partial_\mu L_\nu - \partial_\nu L_\mu \quad (\text{III.7})$$

The Lagrangian for the speedon field is

$$L_o(S) = (\partial_\mu S)(\partial^\mu S), \quad (\text{III.8})$$

and the Lagrangian for the matterfield is

$$L_M(\phi) = (\hat{D}_\mu \phi)^+ (\hat{D}^\mu \phi) - m^2 \phi^+ \phi \quad (\text{III.9})$$

where  $\hat{D}_\mu$  is the overall covariant derivative given by

$$\hat{D}_\mu \phi = D_\mu \phi + i\lambda L_\mu \phi + i(\partial_\mu S)\phi \quad (\text{III.10})$$

$$(\hat{D}_\mu \phi)^+ = D_\mu \phi^+ - i\lambda L_\mu \phi^+ - i(\partial_\mu S)\phi^+ \quad (\text{III.11})$$

with  $D_\mu$  being the geometric covariant derivative, belonging to the graviton.

We also have the following physical dimensions of the coupling constants  $m$  and  $\lambda$ ,

$$[m] = L^{-1} \quad , \quad [\lambda] = M^{-1}L^{-1}Q \quad (\text{III.12})$$

We now get the equations of motion

(i)

$$\begin{aligned}
R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R &= \lambda_0 [F^{\alpha\mu}F_\mu{}^\beta - g^{\alpha\beta}L_o(L_\mu)] \\
&+ 2(\partial^\alpha S)(\partial^\beta S) - g^{\alpha\beta}L_o(S) \\
&+ (\hat{D}^\alpha\phi)^+(\hat{D}^\beta\phi) + (\hat{D}^\beta\phi)^+(\hat{D}^\alpha\phi) \\
&- g^{\alpha\beta}L_M(\phi)
\end{aligned} \tag{III.13}$$

(ii)

$$\lambda_o D_\mu F^{\mu\alpha} + \lambda J^\alpha = 0 \tag{III.14}$$

$$J^\alpha \equiv i \left[ (\hat{D}^\alpha\phi)^+\phi - \phi^+(\hat{D}^\alpha\phi) \right] \tag{III.15}$$

(iii)

$$2D_\mu D^\mu S + D_\mu J^\mu = 0 \tag{III.16}$$

(iv)

$$(\hat{D}_\mu \hat{D}^\mu \phi)^+ + m^2 \phi^+ = 0 \tag{III.17}$$

(v)

$$\hat{D}_\mu \hat{D}^\mu \phi + m^2 \phi = 0 \tag{III.18}$$

From equation (III.14) we find

$$D_\mu J^\mu = 0 \tag{III.19}$$

The interaction gauge group between the electromagnetic field and the matter field is given by

$$\delta_* L_\mu = \partial_\mu \varphi \tag{III.20}$$

$$\delta_* \phi = i\lambda \varphi \cdot \phi \tag{III.21}$$

$$\delta_* \phi^+ = -i\lambda \varphi \cdot \phi^+ \tag{III.22}$$

The gauge group for the speedon field is given by

$$\delta_* S = \text{const.} \quad (\text{III.23})$$

#### IV. Gravity - Electromagnetism - Speedon Field

Here we study the gravitational interaction in the absence of matter, i.e., the Lagrangian (III.3) with  $\phi = 0$ .

We are then left with the Lagrangian

$$L = \sqrt{g}R - 2\lambda_o\sqrt{g}L_o(L_\alpha) - 2\sqrt{g}L_o(S) \quad (\text{IV.1})$$

The equations of motion then read

$$G^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R = \lambda_o [F^{\alpha\mu}F_\mu{}^\beta - g^{\alpha\beta}L_o(L_\alpha)] + 2S^\alpha S^\beta - g^{\alpha\beta}L_o(S) \quad (\text{IV.2})$$

$$D_\mu F^{\mu\alpha} = 0 \quad (\text{IV.3})$$

$$D_\alpha S^\alpha = 0 \quad (\text{IV.4})$$

From Appendix B we find the solutions of these equations in the case of the static Schwarzschild metric and with the speedon field  $S$  as the independent variable as

$$r = -ac_1 \frac{\sinh[c_2 S + c_3]}{\sinh[c_3] \sinh[c_1 S]} \quad (\text{IV.5})$$

$$e^A = \left[ \frac{\sinh[c_3]}{\sinh[c_2 S + c_3]} \right]^2 \quad (\text{IV.6})$$

$$e^B = \frac{c_1^2}{\sinh^2[c_1 S][c_2 \coth[c_2 S + c_3] - c_1 \coth[c_1 S]]^2} \quad (\text{IV.7})$$

$$L = -\frac{b}{a} \frac{c_2}{\mu^2} [\coth[c_2 S + c_3] - \coth[c_3]] \quad (\text{IV.8})$$

$a$  and  $b$  are integration constants and we have the relations

$$\mu^2 = \frac{1}{2}\lambda_o \left( \frac{b}{a} \right)^2 \quad (\text{IV.9})$$

$$c_1^2 = 1 + c_2^2 \quad (\text{IV.10})$$

$$c_2^2 = \mu^2 \sinh^2[c_3] \quad (\text{IV.11})$$



At this time we are only interested in the speedon field. Turning off the electromagnetic interaction means  $\lambda_o = 0$ . This implies  $c_2 = 0$  and  $c_1 = 1$ .

The solutions then become

$$r = -\frac{a}{\sinh[S]} \quad (\text{IV.12})$$

$$e^A = 1 \quad (\text{IV.13})$$

$$e^B = \frac{1}{\cosh^2[S]} \quad (\text{IV.14})$$

or as functions of the radial variable  $r$  we get

$$S = -\ln \left[ \frac{a}{r} + \sqrt{1 + \left( \frac{a}{r} \right)^2} \right] \quad (\text{IV.15})$$

$$e^A = 1 \quad (\text{IV.16})$$

$$e^{-B} = 1 + \left( \frac{a}{r} \right)^2 \quad (\text{IV.17})$$

This is the solution of gravitational interaction with a massless scalar field for the special value  $\alpha = 0$ ; see Appendix C.

From (A.22) we now get an expression for the speed of light

$$\frac{c^2}{c_o^2} = \sqrt{1 + \left( \frac{a}{r} \right)^2} - \frac{a}{r} \quad (\text{IV.18})$$

or written differently

$$\frac{c^2}{c_o^2} = \frac{\frac{r}{a}}{1 + \sqrt{1 + \left( \frac{r}{a} \right)^2}} \quad (\text{IV.19})$$

As  $r \rightarrow 0$  the speed of light goes to zero and as  $r \rightarrow \infty$  the speed of light increases and reaches the reference speed  $c_o$ .

At  $r = a$  one finds  $c = 0.64 c_o$ .

Observe that as  $r$  approaches zero the local physical time becomes very large. There is another interesting observation. The concept of gravitational redshift says that the rates of similar clocks, located at different places in a gravitational field is given by

$$\left[ \frac{N(2)}{N(1)} \right]^2 = \frac{g_{oo}(2)}{g_{oo}(1)} \quad (\text{IV.20})$$

Let  $R$  be a reference radius where the rate of the clock is  $N_R$  and let  $N$  be the rate of a similar clock at the position  $r$ .

For the exterior Schwarzschild metric one gets then

$$\frac{N}{N_R} = \left[ \frac{1 - \frac{a}{r}}{1 - \frac{a}{R}} \right]^{1/2} \quad (\text{IV.21})$$

where  $a$  is the Schwarzschild radius

$$a = 2G_o M \quad (\text{IV.22})$$

For  $r > R$  and  $\frac{a}{R} \ll 1$ , we find

$$\frac{N}{N_R} = 1 + \frac{1}{2} \left[ \frac{a}{R} - \frac{a}{r} \right] + \frac{1}{8} \left[ 3 \left( \frac{a}{R} \right)^2 - 2 \left( \frac{a}{R} \right) \left( \frac{a}{r} \right) - \left( \frac{a}{r} \right)^2 \right] + \dots \quad (\text{IV.23})$$

This applies in particular to similar atomic clocks, one located in Paris and one in Boulder; the one in Boulder ticks faster.

For the gravitational field due to the speedon field alone (IV.16) there is no gravitational redshift. We now compare the speed of light  $c_R$  at the reference radius  $R$  with the speed of light  $c$  at the position  $r$ .

For the integration constant  $a$  in (IV.18) we take the Schwarzschild radius (IV.22).

Then

$$c_R = c_o \left[ \sqrt{1 + \left( \frac{a}{R} \right)^2} - \frac{a}{R} \right]^{1/2} \quad (\text{IV.24})$$

$$c = c_o \left[ \sqrt{1 + \left( \frac{a}{r} \right)^2} - \frac{a}{r} \right]^{1/2} \quad (\text{IV.25})$$

gives the expansion for  $r > R$ ,  $\frac{a}{R} \ll 1$

$$\frac{c}{c_R} = 1 + \frac{1}{2} \left[ \frac{a}{R} - \frac{a}{r} \right] + \frac{1}{8} \left[ \left( \frac{a}{R} \right)^2 - 2 \left( \frac{a}{R} \right) \left( \frac{a}{r} \right) + \left( \frac{a}{r} \right)^2 \right] + \dots \quad (\text{IV.26})$$

This expression agrees with (IV.25) up to first order.

## V. Conclusion

In electrodynamics, elimination of the physical time through dimensional analysis, identifies the speed of light as a dilaton field. In the Lagrange formalism for gravitational interactions with the electromagnetic field and a charged matterfield this point of view introduces a massless scalarfield  $S$  that we call the speedon field. In the absence of the matter field we found solutions for the gravitational-electromagnetic-speedon field equations. If we now turn off the electromagnetic interaction we are left with the speedon field. This is a particular solution of the gravitational interaction with a massless scalar field. The pure speedon field gives an expression for the speed of light. For large values of the radial coordinate the speed of light becomes constant and for small values of the radial coordinate the speed of light goes to zero. This then raises the question about the meaning of a local physical time. The analytic solution in the presence of a matterfield should give some more information. According to Vandyck [2] very accurate redshift magnitude curves should provide information about the presence of a dilaton field. It is interesting to observe that gravitoinal interaction as treated in this paper introduces an “index of refraction” for the Universe.

## Appendix A: Scaling the Electromagnetic Field

Electrodynamics is described by a vector field  $\{A_\mu\}$ , the so-called vectorpotential, which belongs to the restmass zero, spin one representation of the Lorentz group.

The physical dimension of  $A_\mu$  is

$$[A_\mu] = ML^2T^{-2}Q^{-1} \quad (\text{A.1})$$

with  $c$  denoting the speed of light,  $t$  the physical time and

$$\{A_\mu\} = (A_o, \mathbf{A}) \quad (\text{A.2})$$

the electric field is given by

$$\mathbf{E} = \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \text{grad } A_o \quad (\text{A.3})$$

and the magnetic field by

$$\mathbf{B} = -\frac{1}{c} \text{curl } \mathbf{A} \quad (\text{A.4})$$

These fields have the physical dimensions

$$[E] = MLT^{-2}Q^{-1} \quad (\text{A.5})$$

$$[B] = MT^{-1}Q^{-1} \quad (\text{A.6})$$

We now introduce the geometric time  $x^o$ , which is related to the physical time  $t$  by

$$x^o = ct \quad (\text{A.7})$$

and the scaled field  $L_\mu$  through

$$L_\mu = \frac{1}{c^2} A_\mu. \quad (\text{A.8})$$

$L_\mu$  has the physical dimension

$$[L_\mu] = MQ^{-1} \quad (\text{A.9})$$

which does not depend on the physical time. Maxwell's equations follow from a variational principle with the Lagrangian

$$L_o(L_\mu) = \frac{1}{4\pi K_o} \cdot \frac{1}{4} F_{\mu\nu} F^{\nu\mu} \quad (\text{A.10})$$

where

$$K_o = \frac{K}{c^2}, \quad (\text{A.11})$$

with  $K$  being the Coulomb constant and

$$F_{\mu\nu} \equiv \partial_\mu L_\nu - \partial_\nu L_\mu \quad (\text{A.12})$$

The physical dimensions are

$$[K_o] = MLQ^{-2} \quad (\text{A.13})$$

$$[L_o] = ML^{-3} \quad (\text{A.14})$$

The Lagrangian  $L_o$  is a mass density.

Through scaling with the help of the speed of light electrodynamics is thus formulated in such a way that it does not depend on physical time. This now opens the possibility that the speed of light could be a function on Minkowski space [1]. With

$$\{L_\mu\} = (L_o, \mathbf{L}) \quad (\text{A.15})$$

and  $c$  being the local speed of light, we can now retrieve the local quantities

$$A_\mu = c^2 L_\mu \quad (\text{A.16})$$

$$\mathbf{E} = c^2 \left[ \frac{\partial \mathbf{L}}{\partial x^o} - \text{grad } L_o \right] \quad (\text{A.17})$$

$$\mathbf{B} = -c \text{ curl } \mathbf{L} \quad (\text{A.18})$$

$$t = \frac{1}{c} x^o \quad (\text{A.19})$$

The physical time thus becomes a local concept.

The scaling can be written as

$$L_\mu = \frac{c_o^2}{c^2} \frac{1}{c_o^2} A_\mu \quad (\text{A.20})$$

with  $c_o$  as a reference speed of light. Then

$$L_\mu = e^{-S} \frac{1}{c_o^2} A_\mu \quad (\text{A.21})$$

where

$$S = \ln \left( \frac{c^2}{c_o^2} \right) \quad (\text{A.12})$$

$S$  is thus a dilation field [2] and belongs to the restmass zero, spin zero representation of the Lorentz group. Since  $S$  is related to the speed of light we call it the speedon field.

## Appendix B: The Schwarzschild metric and the solutions

The Schwarzschild metric is given by

$$ds^2 = e^A (dx^o)^2 - e^B (dr)^2 - r^2 \{d\theta^2 + \sin^2 \theta d\varphi^2\} \quad (\text{B.1})$$

For the static case  $A = A(r), B = B(r)$ . With  $g = -\text{Det}(g_{\alpha\beta})$  we find

$$\sqrt{g} = e^{\frac{1}{2}(A+B)} r^2 \sin \theta \quad (\text{B.2})$$

and the Einstein tensor [3]

$$G^\alpha{}_\beta = g^{\alpha\gamma} [R_{\gamma\beta} - \frac{1}{2} g_{\gamma\beta} \cdot R] \quad (\text{B.3})$$

reads

$$G^0{}_0 = -\frac{1}{r^2} \frac{d}{dr} [r(e^{-B} - 1)] \quad (\text{B.4})$$

$$G^1{}_1 = G^0{}_0 - \frac{1}{r} e^{-B} \frac{d}{dr} (A + B) \quad (\text{B.5})$$

$$G^2{}_2 = \frac{1}{2r} \frac{d}{dr} [r^2 G^1{}_1] - \frac{1}{4} e^{-B} \frac{dA}{dr} \frac{dB}{dr} (A + B) \quad (\text{B.6})$$

$$G^3{}_3 = G^2{}_2 \quad (\text{B.7})$$

All other components vanish.

We now introduce the auxiliary functions

$$f = e^{\frac{1}{2}(A+B)} \quad (\text{B.8})$$

$$h = r e^{\frac{1}{2}(A-B)} \quad (\text{B.9})$$

Then

$$e^A = \frac{1}{r} f h \quad (\text{B.10})$$

$$e^B = \frac{r f}{h} \quad (\text{B.11})$$

and

$$\sqrt{g} = r^2 f \sin \theta \quad (\text{B.12})$$

The relevant components of the Einstein tensor then become

$$G^0_0 = -\frac{1}{r^2} \frac{d}{dr} \left[ \frac{h}{f} - r \right] \quad (\text{B.13})$$

$$G^1_1 = G^0_0 - 2 \frac{h \frac{df}{dr}}{r^2 f^2} \quad (\text{B.14})$$

$$G^2_2 = \frac{1}{2r} \frac{d}{dr} [r^2 G^1_1] - \frac{1}{2} \frac{h \frac{df}{dr}}{r f^2} \frac{d}{dr} \ln \left( \frac{f h}{r} \right) \quad (\text{B.15})$$

With  $(L_\alpha) = (L, 0, 0, 0)$  we find the Lagrangian for the electromagnetic field as

$$L_0(L_\alpha) = \frac{1}{2f^2} \frac{dL}{dr} \frac{dL}{dr} \quad (\text{B.16})$$

For the speedon field the Lagrangian reads

$$L_o(S) = -\frac{h}{r f} \frac{dS}{dr} \frac{dS}{dr} \quad (\text{B.17})$$

The equations of motion then become

$$G^0_0 = \frac{1}{2} \lambda_o \frac{1}{f^2} \frac{dL}{dr} \frac{dL}{dr} + \frac{h}{r f} \frac{dS}{dr} \frac{dS}{dr} \quad (\text{B.18})$$

$$G^1_1 = \frac{1}{2} \lambda_o \frac{1}{f^2} \frac{dL}{dr} \frac{dL}{dr} - \frac{h}{r f} \frac{dS}{dr} \frac{dS}{dr} \quad (\text{B.19})$$

$$G^2_2 = -\frac{1}{2} \lambda_o \frac{1}{f^2} \frac{dL}{dr} \frac{dL}{dr} + \frac{h}{r f} \frac{dS}{dr} \frac{dS}{dr} \quad (\text{B.20})$$

$$\frac{d}{dr} \left[ \frac{r^2}{f} \frac{dL}{dr} \right] = 0 \quad (\text{B.21})$$

$$\frac{d}{dr} \left[ rh \frac{dS}{dr} \right] = 0 \quad (\text{B.22})$$

We now find the following system of independent equations

$$\frac{dL}{dr} = b \frac{f}{r^2} \quad (\text{B.23})$$

$$\frac{dS}{dr} = \frac{a}{rh} \quad (\text{B.24})$$

$$\frac{1}{f} \frac{df}{dr} = r \frac{dS}{dr} \frac{dS}{dr} \quad (\text{B.25})$$

$$\frac{dh}{dr} = f \left[ 1 - \frac{1}{2} \lambda_o b^2 \frac{1}{r^2} \right] \quad (\text{B.26})$$

Introducing the dimensionless quantities

$$r = ax \quad , \quad h = ak \quad (\text{B.27})$$

and the abbreviation  $' \equiv \frac{d}{dx}$ , we find the equation of motion to read

$$L' = \frac{b}{a} \frac{1}{x^2} f \quad (\text{B.28})$$

$$S' = \frac{1}{xk} \quad (\text{B.29})$$

$$\frac{f'}{f} = x S' S' \quad (\text{B.30})$$

$$k' = f \left[ 1 - \frac{1}{2} \lambda_o \left( \frac{b}{a} \right)^2 \cdot \frac{1}{x^2} \right] \quad (\text{B.31})$$

As in [9, 10] we look upon the speedon field as the independent variable and denote by  $\cdot \equiv \frac{d}{dS}$ . The equations of motion then read

$$\dot{L} = \frac{b}{a} \frac{\dot{x}}{x^2} f \quad (\text{B.32})$$

$$\dot{x} \dot{f} = x f \quad (\text{B.33})$$

$$\dot{x} = x k \quad (\text{B.34})$$

$$\dot{k} = \dot{x} f \left[ 1 - \mu^2 \frac{1}{x^2} \right] \quad (\text{B.35})$$



with

$$\mu^2 = \frac{1}{2} \lambda_o \left( \frac{b}{a} \right)^2.$$

The boundary conditions we impose are

$$S \rightarrow 0 \quad \Rightarrow \quad f \rightarrow 1, x \rightarrow \infty, \frac{k}{x} \rightarrow 1, L \rightarrow 0, \quad (\text{B.36})$$

implying asymptotic flatness, and the vanishing of the speedon field at infinity.

To solve these equations we introduce the auxiliary function  $F(S)$  through

$$\frac{\dot{x}}{x} = F \quad (\text{B.37})$$

Then from (B.33) we get

$$\frac{\dot{f}}{f} = \frac{1}{F} \quad (\text{B.38})$$

This leads to

$$x = x_o e^{\int F dS} \quad (\text{B.39})$$

$$f = f_o e^{\int \frac{1}{F} dS} \quad (\text{B.40})$$

Equations (B.34) and (B.35) then become

$$k = F \quad (\text{B.41})$$

$$\dot{F} = F x_o f_o e^{\int [F + \frac{1}{F}] dS} \left[ 1 - \mu^2 \frac{1}{x_o^2} e^{-2 \int F dS} \right] \quad (\text{B.42})$$

With

$$\alpha \equiv x_o f_o, \quad \beta = \mu^2 f_o \frac{1}{x_o} \quad (\text{B.43})$$

equation (B.42) reads

$$\frac{\dot{F}}{F} = \alpha e^{\int [\frac{1}{F} + F] dS} - \beta e^{\int [\frac{1}{F} - F] dS} \quad (\text{B.44})$$

Differentiating this equation and substituting it we get

$$\frac{d}{dS} \left[ \frac{\dot{F}}{F} \right] = \frac{\dot{F}}{F^2} + F \left\{ \alpha e^{\int [\frac{1}{F} + F] dS} + \beta e^{\int [\frac{1}{F} - F] dS} \right\} \quad (\text{B.45})$$

Adding and subtracting equations (B.44) and (B.45) and using the abbreviations

$$X = \frac{\dot{F}}{F} + F + \frac{1}{F} \quad (\text{B.46})$$

$$Y = \frac{\dot{F}}{F} - F + \frac{1}{F} \quad (\text{B.47})$$

results in

$$2\alpha e \int [\frac{1}{F} + F] dS = \frac{1}{F} \dot{X} \quad (\text{B.48})$$

$$2\beta e \int [\frac{1}{F} - F] dS = \frac{1}{F} \dot{Y} \quad (\text{B.49})$$

and

$$2F = X - Y \quad (\text{B.50})$$

$$2 \left[ \frac{\dot{F}}{F} + \frac{1}{F} \right] = X + Y \quad (\text{B.51})$$

Taking the logarithmic derivative of equations (B.48) and (B.49) we get

$$\frac{1}{F} + F = -\frac{\dot{F}}{F} + \frac{\ddot{X}}{\dot{X}} \quad (\text{B.52})$$

$$\frac{1}{F} - F = -\frac{\dot{F}}{F} + \frac{\ddot{Y}}{\dot{Y}} \quad (\text{B.53})$$

resulting in the equations

$$\ddot{X} = X \dot{X} \quad (\text{B.54})$$

$$\ddot{Y} = Y \dot{Y} \quad (\text{B.55})$$

Observe that from equations (B.10) and (B.11) we have the relations

$$\dot{X} = 2F^2 e^B \quad (\text{B.56})$$

$$\dot{Y} = 2\mu^2 e^A \quad (\text{B.57})$$

Asymptotic flatness then demands

$$\dot{Y}(0) = 2\mu^2 \quad (\text{B.58})$$

and that both  $\dot{X}$  and  $\dot{Y}$  are positive.

The solutions of (B.54) and (B.55) that are compatible with the boundary conditions (B.36), (B.58) are given by

$$X = -2c_1 \coth[c_1 S] \quad (\text{B.59})$$

$$Y = -2c_2 \coth[c_2 S + c_3] \quad (\text{B.60})$$

From (B.50) and (B.51) we then get

$$F = c_2 \coth[c_2 S + c_3] - c_1 \coth[c_1 S] \quad (\text{B.61})$$

and the condition

$$c_1^2 = 1 + c_2^2 \quad (\text{B.62})$$

The condition (B.58) then becomes

$$c_2^2 = \mu^2 \sinh^2[c_3] \quad (\text{B.63})$$

We then find the solutions of (B.32) to (B.35)

$$x = -\frac{c_1}{\sinh[c_3]} \frac{\sinh[c_2 S + c_3]}{\sinh[c_1 S]} \quad (\text{B.64})$$

$$f = \frac{c_1 \sinh[c_3]}{\sinh[c_2 S + c_3] \sinh[c_1 S] \{c_1 \coth[c_1 S] - c_2 \coth[c_2 S + c_3]\}} \quad (\text{B.65})$$

$$k = c_2 \coth[c_2 S + c_3] - c_1 \coth[c_1 S] \quad (\text{B.66})$$

$$L = -\frac{b}{a} \frac{c_2}{\mu^2} [\coth[c_2 S + c_3] - \coth[c_3]] \quad (\text{B.67})$$

From (B.56) and (B.57) we get

$$e^A = \left[ \frac{\sinh[c_3]}{\sinh[c_2 S + c_3]} \right]^2 \quad (\text{B.68})$$

$$e^B = \frac{c_1^2}{\sinh^2[c_1 S] [c_2 \coth[c_2 S + c_3] - c_1 \coth[c_1 S]]^2} \quad (\text{B.69})$$

## Appendix C: The Massless Scalarfield

Gravitational interaction with a massless scalarfield alone is given by setting  $\lambda_o = 0$  in (IV.1). This corresponds to setting  $\mu = 0$  in (B.57). The equations (B.59) and (B.60) then are replaced by

$$X = -2c_1 \coth[c_1 S] \quad (\text{C.1})$$

$$Y = 2\alpha \quad (\text{C.2})$$

where  $\alpha$  is a constant.

From (B.50) and (B.51) we then get

$$F = -\alpha - c_1 \coth[c_1 S] \quad (\text{C.3})$$

and the condition

$$c_1^2 = 1 + \alpha^2 \quad (\text{C.4})$$

The complete solution [4] is then given by

$$x = -c_1 e^{-\alpha S} \frac{1}{\sinh[c_1 S]} \quad (\text{C.5})$$

$$f = c_1 e^{\alpha S} \frac{1}{c_1 \cosh[c_1 S] + \alpha \sinh[c_1 S]} \quad (\text{C.6})$$

$$k = -\alpha - c_1 \coth[c_1 S] \quad (\text{C.7})$$

$$e^A = e^{2\alpha S} \quad (\text{C.8})$$

$$e^B = \left[ \frac{c_1}{c_1 \cosh[c_1 S] + \alpha \sinh[c_1 S]} \right]^2 \quad (\text{C.9})$$

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